ESTIMATION OF MEAN WHEN POPULATION VARIANCE IS KNOWN

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SUMMARY

In this paper we have suggested a class of estimators for the population mean $\bar{x}$ when the population variance is known and studied its properties under large sample approximation.

INTRODUCTION

Let $x_1, x_2, \ldots, x_n$ be random sample of size n from a population with unknown mean $\bar{x}$ and known variance σ^2. Assuming the sample mean $\bar{x}$ not near zero, Upadhyaya and Srivastava [1] proposed and estimator

$$\hat{\bar{x}}_1=\bar{x}+\frac{6^2}{n\bar{x}} \tag{1}$$

and obtained the bias and mean—squared error (M.S.E.) to order $o(n^{-2})$ as

$$\text{Bias}\ (\hat{\bar{x}}_1)=\frac{\sigma^2}{n\bar{x}}\left(1+\frac{\sigma^2}{n\bar{x}^2}\right) \tag{2}$$

$$\text{MSE}\ (\hat{\bar{x}}_1)=\frac{\sigma^2}{n}\left(1-\frac{\sigma^2}{n\bar{x}^2}\right) \tag{3}$$

The purpose of this paper is to propose a class of the estimators of the mean when the population variance is known. In section 2 a class of estimators of the mean is proposed and its large sample approximations discussed in section 3.

ESTIMATOR

Assuming the population variance σ^2 to be known, we propose the class of estimators for the mean $\bar{x}$ as

$$\hat{\bar{x}}_k = \bar{x} + K \frac{\sigma^2 \bar{x}}{n \bar{x}^2 + \sigma^2} \tag{4}$$

where K is the characterizing scalar to be chosen suitably.

The bias and MSE of $\hat{\bar{x}}_k$ are

$$\text{Bias } (\hat{\bar{x}}_k) = K \sigma^2 E \left(\frac{\bar{x}}{n \bar{x}^2 + \sigma^2} \right) \tag{5}$$

$$\text{MSE } (\hat{\bar{X}}_k) = E(\bar{x} - \bar{X})^2 + K^2 \sigma^4 E \left(\frac{\bar{x}}{n \bar{x}^2 + \sigma^2} \right)^2$$

$$+ 2K \sigma^2 E \left(\frac{\bar{x}^2}{n \bar{x}^2 + \sigma^2} \right) - 2k \bar{X} \sigma^2 E \left(\frac{\bar{x}}{n \bar{x}^2 + \sigma^2} \right). \tag{6}$$

The MSE$(\hat{\bar{X}}_k)$ is minimum when

$$K = \frac{\bar{X} E\left(\frac{\bar{x}}{n \bar{x}^2 + \sigma^2} \right) - E\left(\frac{\bar{x}^2}{n \bar{x}^2 + \sigma^2} \right)}{\sigma^2 E \left(\frac{\bar{x}}{n \bar{x}^2 + \sigma^2} \right)^2} = K_{min} \text{ (say)} \tag{7}$$

In the expressions of bias and MSE, we observe that the determination of the expectations are difficult as the expectation of the

$$\bar{x} \, (n \, \bar{x}_2 + 0^2)^{-1}, \; \bar{x}^2 \, (n \, \bar{x}^2 + \sigma^2)^{-2} \text{ and } \bar{x}^2 \, (n\bar{x}^2 + \sigma^2)^{-1}$$

are mathematically intractable. We have, therefore, derived large sample approximations for these expectations.

LARGE SAMPLE APPROXIMATIONS

To investigate the large sample properties of the estimator $\hat{\bar{X}}_{K'}$ we write

$$\bar{x} = \bar{X} + \epsilon$$

where ϵ is the order of $0\,(n^{-\frac{1}{2}})$ with $E(\epsilon) = 0$. We choose n large enough so that

$$\left| \frac{\epsilon}{\bar{X}} \right| < 1.$$

We have

$$\frac{\bar{x}}{n \, \bar{x}^2 + \sigma^2} = \frac{1}{n\bar{X}} \left(1 + \frac{\epsilon}{\bar{X}} \right) \left(1 + \frac{2\epsilon}{\bar{X}} + \frac{\epsilon^2}{\bar{X}^2} + \frac{\sigma^2}{n \, \bar{X}^2} \right)^{-1}$$

Expanding the r.h.s. and taking the expectations of both the sides and retaining the term of order $0(n^{-2})$, we find

$$E\left(\frac{\bar{x}}{n\,\bar{x}^2+\sigma^2}\right)=\frac{1}{n\,\bar{X}} \tag{8}$$

Similarly, to order $o(n^{-2})$ we can obtain

$$E\left(\frac{\bar{x}}{n\,\bar{x}^2+\sigma^2}\right)^2=\frac{1}{n^2\,\bar{X}^2} \tag{9}$$

$$E\left(\frac{\bar{x}^2}{n\,\bar{x}^2+\sigma^2}\right)=\frac{1}{n}\left(1-\frac{\sigma^2}{n\,\bar{X}^2}\right) \tag{10}$$

From (5), (6), (8), (9) and (10) we get the bias and MSE of the estimator $\hat{\bar{X}}_k$ to order $o(n^{-2})$ as

$$\text{Bias }(\hat{\bar{X}}_k)=K\frac{\sigma^2}{n\bar{X}} \tag{11}$$

$$\text{MSE }(\hat{\bar{X}}_k)=\frac{\sigma^2}{n}\left[1+K(K-2)\frac{\sigma^2}{n\,\bar{X}^2}\right] \tag{12}$$

If we minimize MSE $(\hat{\bar{X}}_k)$ w.r.t. K, we find

$$K=1$$

Then the estimator $\hat{\bar{X}}_k$ becomes

$$\hat{\bar{X}}=\bar{x}+\frac{\sigma^2\,\bar{x}}{n\,\bar{x}^2+\sigma^2} \tag{13}$$

Thus the bias and MSE of the estimator $\hat{\bar{X}}$ are

$$\text{Bias }(\hat{\bar{X}})=\frac{\sigma^2}{n\,\bar{X}} \tag{14}$$

$$\text{MSE }(\hat{\bar{X}})=\frac{\sigma^2}{n}\left(1-\frac{\sigma^2}{n\,\bar{X}^2}\right) \tag{15}$$

From (2), (3), (14) and (15) we have

Bias $(\hat{\bar{X}})<$ Bias $(\hat{\bar{X}}_1)$ and MSE $(\hat{\bar{X}})=$ MSE $(\hat{\bar{X}}_1)$

Therefore our estimator $\hat{\bar{X}}$ is superior to $\hat{\bar{X}}_1$.

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REFERENCE

[1] Upadhyaya, L.N. and Srivastava, S.R. : An efficient estimators of mean when population variance is known, *Jour. Ind. Soc. Agri. Stat.*, Vol. XXVIII, No. 1, 1976, pp. 99-102.